A Bound on Mean-Square Estimation Error Accounting for System Model Mismatch

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Abstract In typical array processing problems, the signal observation is a function of the parameter set to be estimated as well as some background system model assumed known. The modeled background could differ from the true one, leading to biased estimates even at high signal-to-noise ratio (SNR). To analyze this system model mismatch problem, a Ziv-Zakai-type lower bound on the mean-square error is developed based on the mismatched likelihood ratio test (MLRT). At high SNR, the bound incorporates the increase in mean-square error due to estimation bias; at low SNR, it includes the threshold effect due to estimation ambiguity. The kernel of the bound's evaluation is the error probability associated with the MLRT. A closed-form expression for this error probability is derived under a data model typical of the array problem assuming random signal embedded in random noise, both of which can be spatially correlated and potentially mismatched. The development is applied to plane-wave bearing estimation with array shape mismatch and matched-field source localization with channel parameter mismatch. Examples demonstrate that the developed bound describes the simulations of the maximum likelihood estimate well, including the sidelobe-introduced threshold behavior and the bias at high SNR.

[1] Y. Rockah and P.M. Schultheiss, "Array Shape Calibration Using Sources in Unknown Locations—Part I: Far-field Sources," IEEE Trans. Acoustics, Speech, and Signal Processing, Vol. 35, pp. 286–299, Mar. 1987.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **20 DEC 2004**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**
4. TITLE AND SUBTITLE **A Bound on Mean-Square Estimation Error Accounting for System Model Mismatch**		5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **RD Instruments; MIT Lincoln Laboratory**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S) 11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also, ADM001741 Proceedings of the Twelfth Annual Adaptive Sensor Array Processing Workshop, 16-18 March 2004 (ASAP-12, Volume 1)., The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**UU**	**15**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

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The 12th Annual Workshop on Adaptive Sensor Array Processing

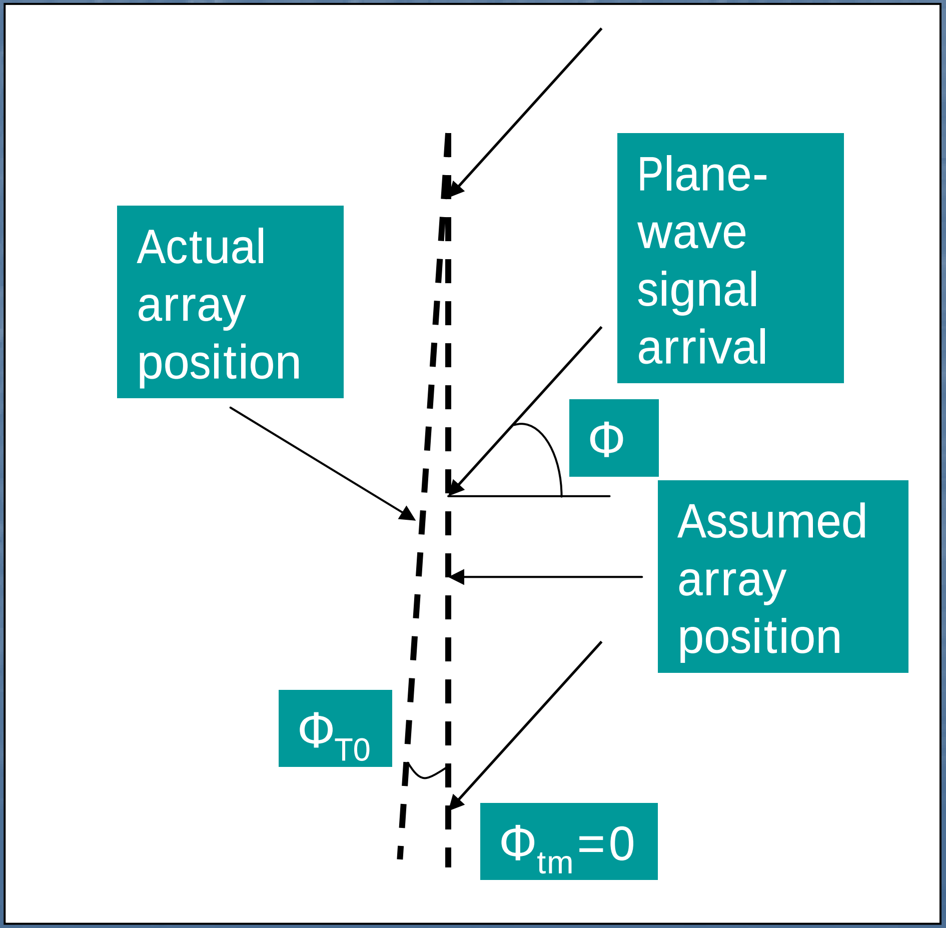
March 16, 2004

General Parameter Estimation System Model

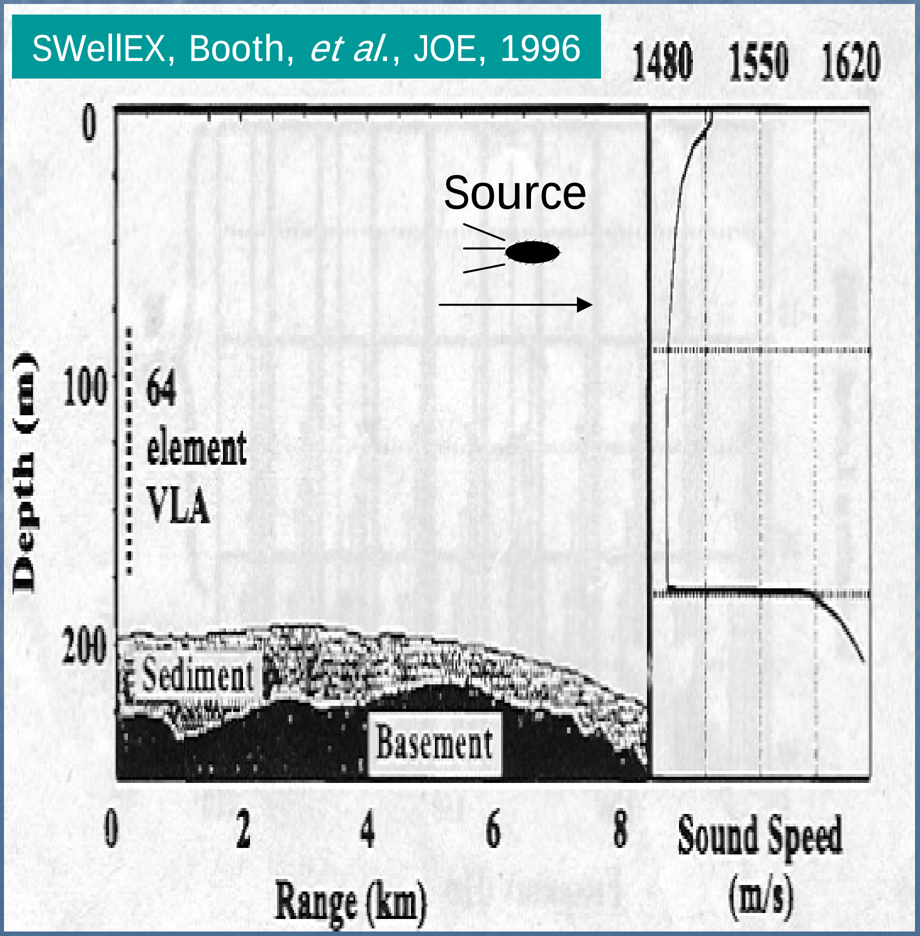
- $p(\mathbf{r}(\boldsymbol{\theta},B); \mathcal{M}(\boldsymbol{\theta},B))$ - system probability model
  - $\mathbf{r}(\cdot)$: random observation vector
    - $\boldsymbol{\theta}$: source/channel/system parameter vector to be estimated
    - B: any other background source/system/channel parameter vector, not included in $\boldsymbol{\theta}$, but embedded in $\mathbf{r}$
  - $\mathcal{M}(\cdot)$: collection of constant scalars, vectors, or matrices, directly defining the pdf, constructed from the moments, e.g., the mean and covariance matrix
- In reality - $p(\mathbf{r}(\boldsymbol{\theta},B_0); \mathcal{M}(\boldsymbol{\theta},B_m))$
    - B_0: true background set
    - B_m: modeled background set
- System model mismatch: $B_m \neq B_0$

Example Scenarios with System Model Mismatch

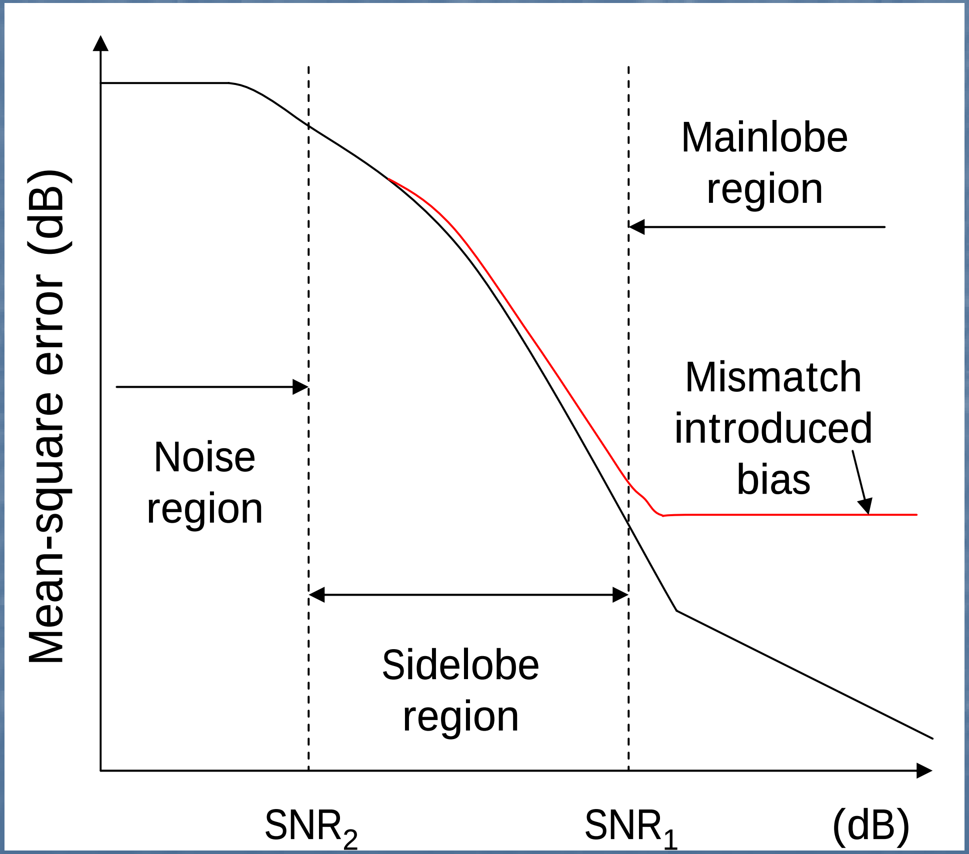
Sensor array perturbation in plane-wave source bearing estimation



Environmental model mismatch in matched-field source localization



Typical Performance Behavior in Bearing Estimation/Passive Source Localization



(re Bayesian Maximum Likelihood Estimate)

- Without system model mismatch
  - High SNR: small mean-square error due to mainlobe peak distortion
  - Intermediate SNR: significantly increased error due to ambiguity sidelobes
  - Low SNR: dominated by noise, subject to the *a priori* parameter distribution
  - Performance analysis tool
    - Cramer-Rao bound
    - Ziv-Zakai bound
- With system model mismatch
  - High SNR: increased error due to mismatch-introduced bias
  - Performance analysis tool
    - Pseudo Cramer-Rao bound
    - Modified Ziv-Zakai bound

Ziv-Zakai bound (ZZB) is derived based on the probability of deciding correctly between two hypotheses corresponding to two parameter values:

H_0: $\mathbf{r}_l(f_m) \sim \mathcal{N}(\mathbf{0}, K_{\mathbf{r}}(f_m,\theta))$

H_1: $\mathbf{r}_l(f_m) \sim \mathcal{N}(\mathbf{0}, K_{\mathbf{r}}(f_m,\theta+\delta))$

$\mathbf{r}_l$: l-th snapshot
f_m: m-th frequency component
$\mathcal{N}()$: Gaussian distribution
$K_{\mathbf{r}}$: Covariance matrix
δ : parameter perturbation

- Scalar parameter bound

$$\varepsilon^2 \geq \int_0^\infty \delta \cdot \left(\int min[p(\theta),p(\theta+\delta)] \cdot P_e(\theta, \theta+\delta) d\theta\right) \cdot d\delta$$

  - ε^2: mean-square error
  - $p(\theta)$: prior parameter distribution pdf
  - $P_e(\theta, \theta+\delta)$: minimum achievable probability of error associated with the likelihood ratio test (LRT)

- Random Gaussian signal embedded in spatially-white Gaussian noise

$$LRT = \sum_{m=1}^{M} \sum_{l=1}^{L} |\mathbf{r}_l^\dagger(f_m)\mathbf{g}(f_m,\theta)|^2 - \sum_{m=1}^{M} \sum_{l=1}^{L} |\mathbf{r}_l^\dagger(f_m)\mathbf{g}(f_m,\theta+\delta)|^2$$

  - $\mathbf{g}(f_m,\theta)$: channel transfer function (Green's function)

A modified Ziv-Zakai bound has been developed for performance analysis under system model mismatch

- Mismatched likelihood ratio test (MLRT)

$$\Sigma_{m=1}^{M} \Sigma_{l=1}^{L} |\mathbf{r}_l^{\dagger}(f_m, B_0)\mathbf{g}(f_m,\theta, B_m)|^2 - \Sigma_{m=1}^{M} \Sigma_{l=1}^{L} |\mathbf{r}_l^{\dagger}(f_m, B_0)\mathbf{g}(f_m,\theta+\delta, B_m)|^2$$

- The modified ZZB is given by

$$\varepsilon^2 \geq \int_0^{\infty}\delta\cdot(\int \mathit{min}[p(\theta),p(\theta+\delta)]\cdot P_{e\text{-}mis}(\theta, \theta+\delta)d\theta)\cdot d\delta$$

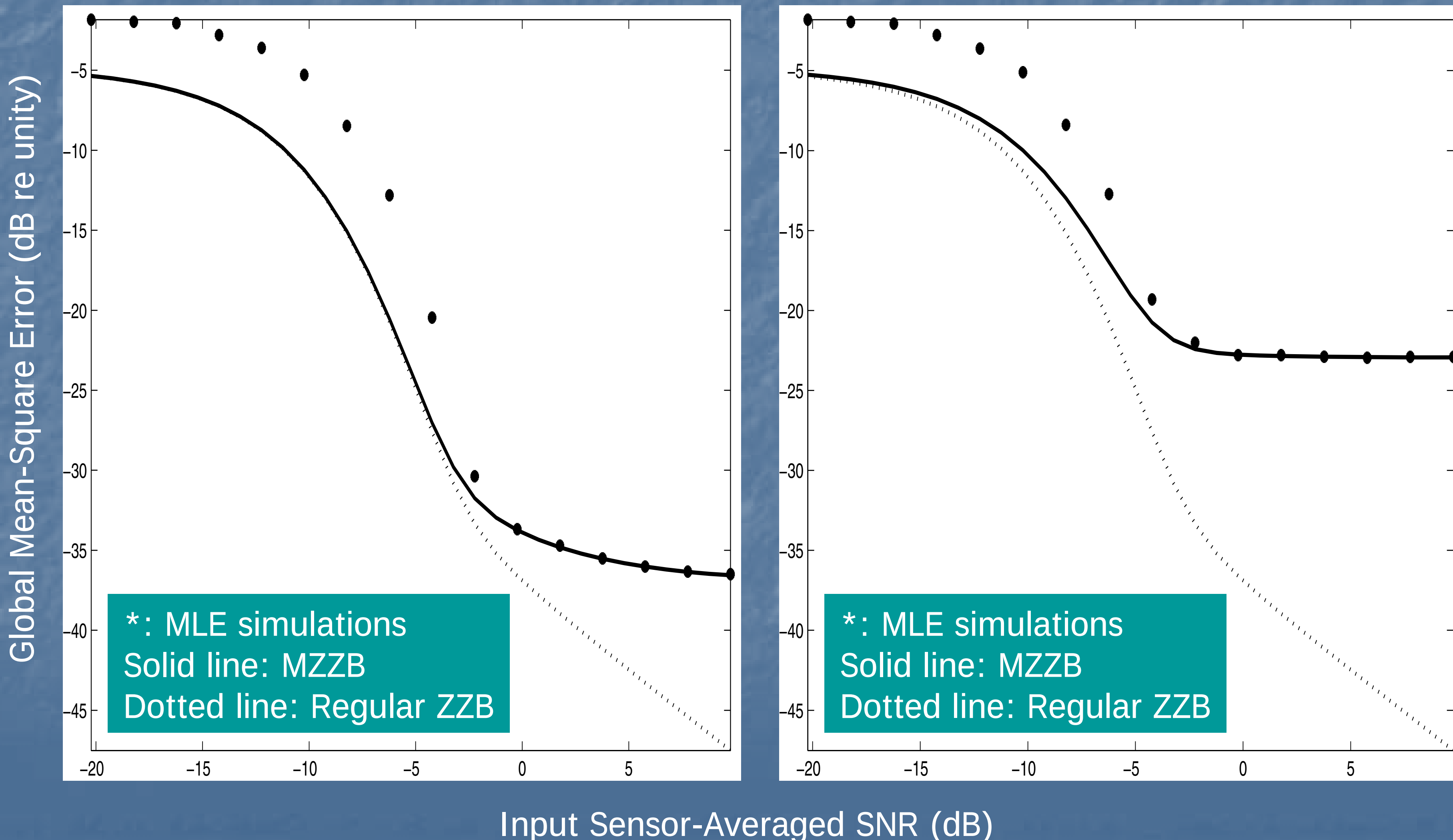
    - $P_{e\text{-}mis}(\theta, \theta+\delta)$: minimum achievable probability of error associated with the MLRT
    - Square-root at high SNR defines an upper bound for estimation bias

- A closed-form expression of $P_{e\text{-}mis}(\theta, \theta+\delta)$ has been derived for a general class of data model
    - Multiple-frequency, multiple-snapshot
    - Both signal and noise can be spatially correlated
    - Both signal and noise can be potentially mismatched

Bearing estimation mean-square error

Array tilt angle mismatched by 1˚

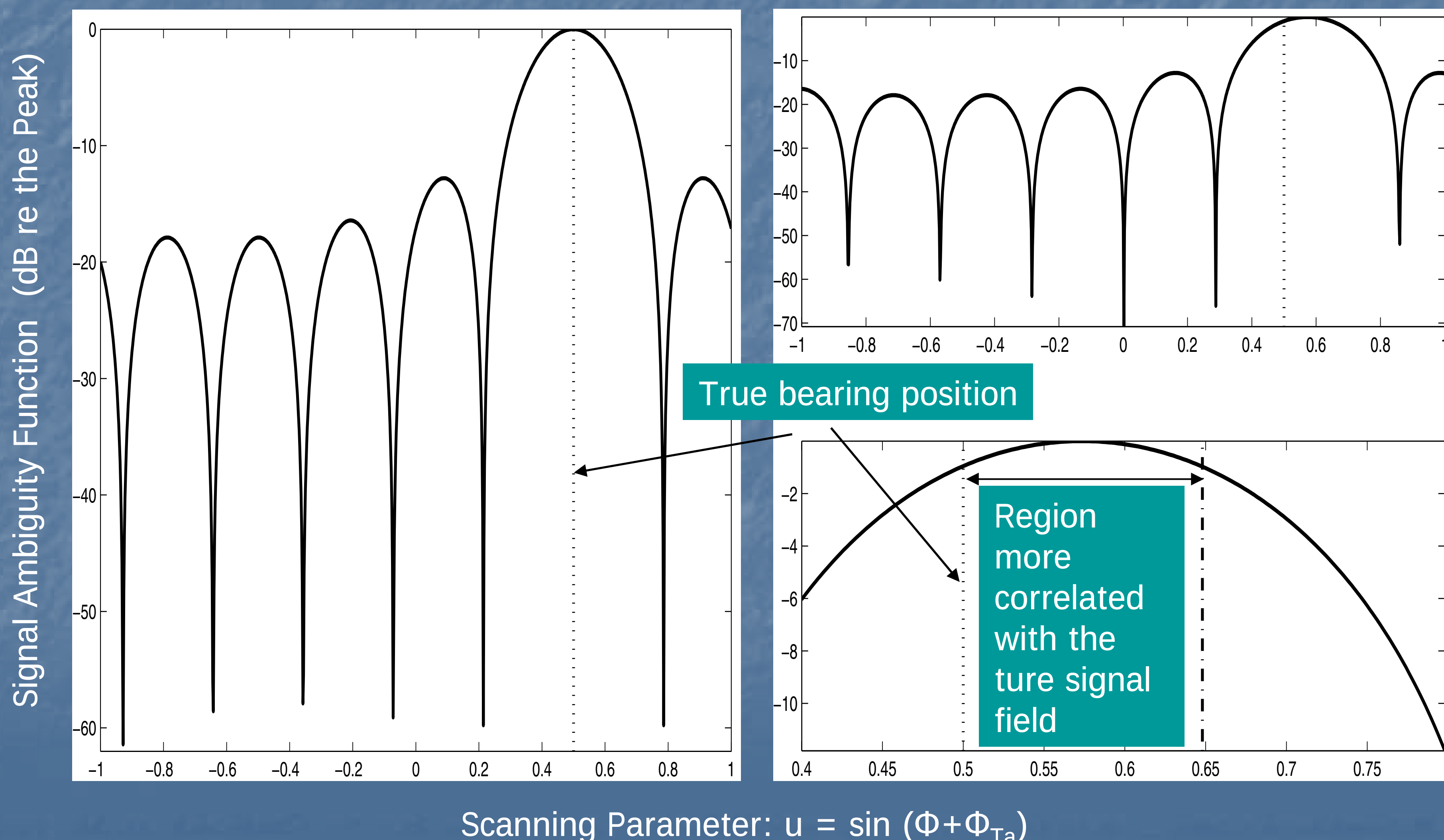
Array tilt angle mismatched by 5˚



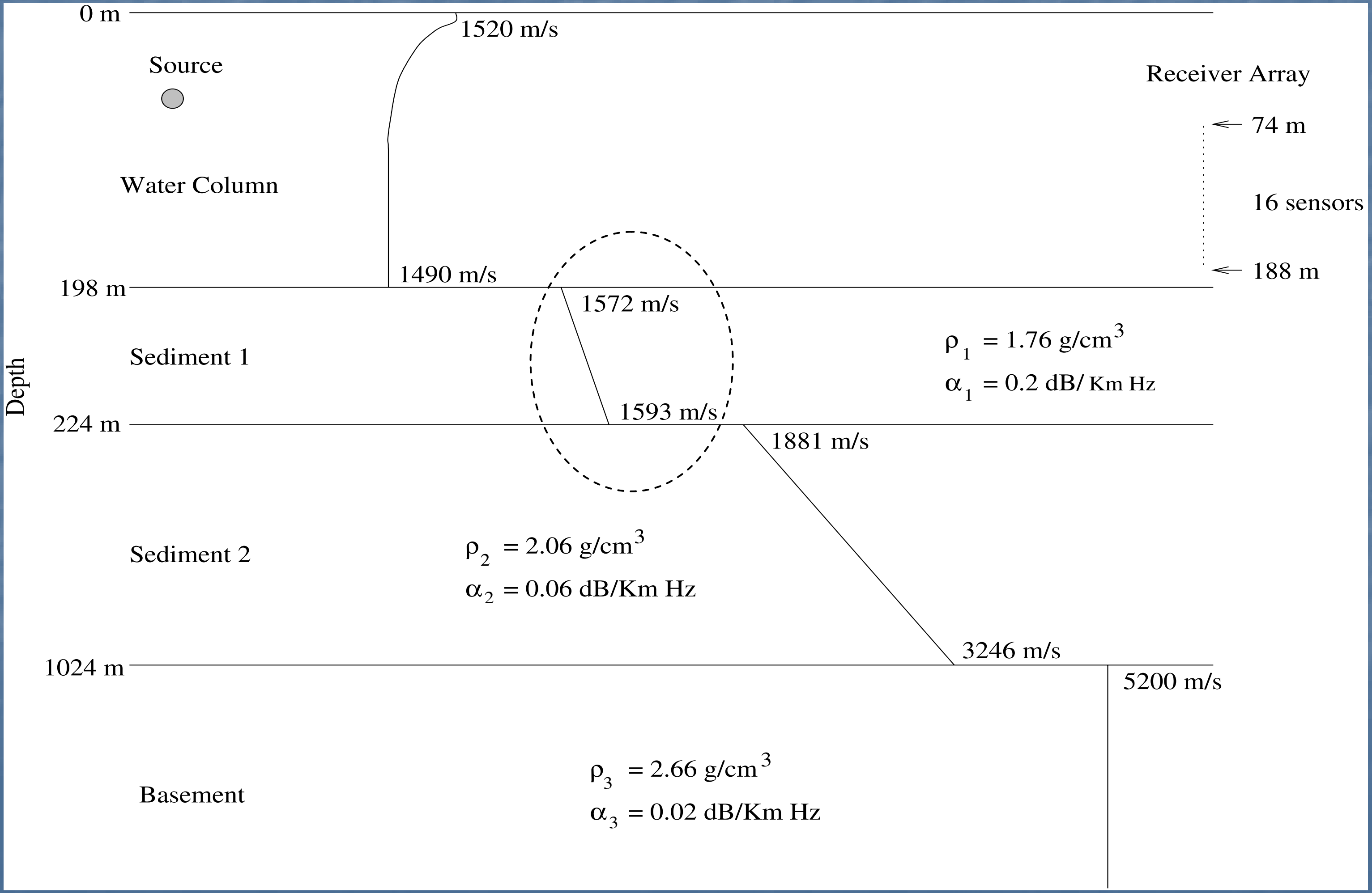
Ambiguity function in bearing estimation

No mismatch

Array tilt angle mismatched by 5˚



Matched-field methods achieve performance improvement over the plane-wave approach by exploiting full field signal propagation, henceforth more sensitive to environmental mismatch

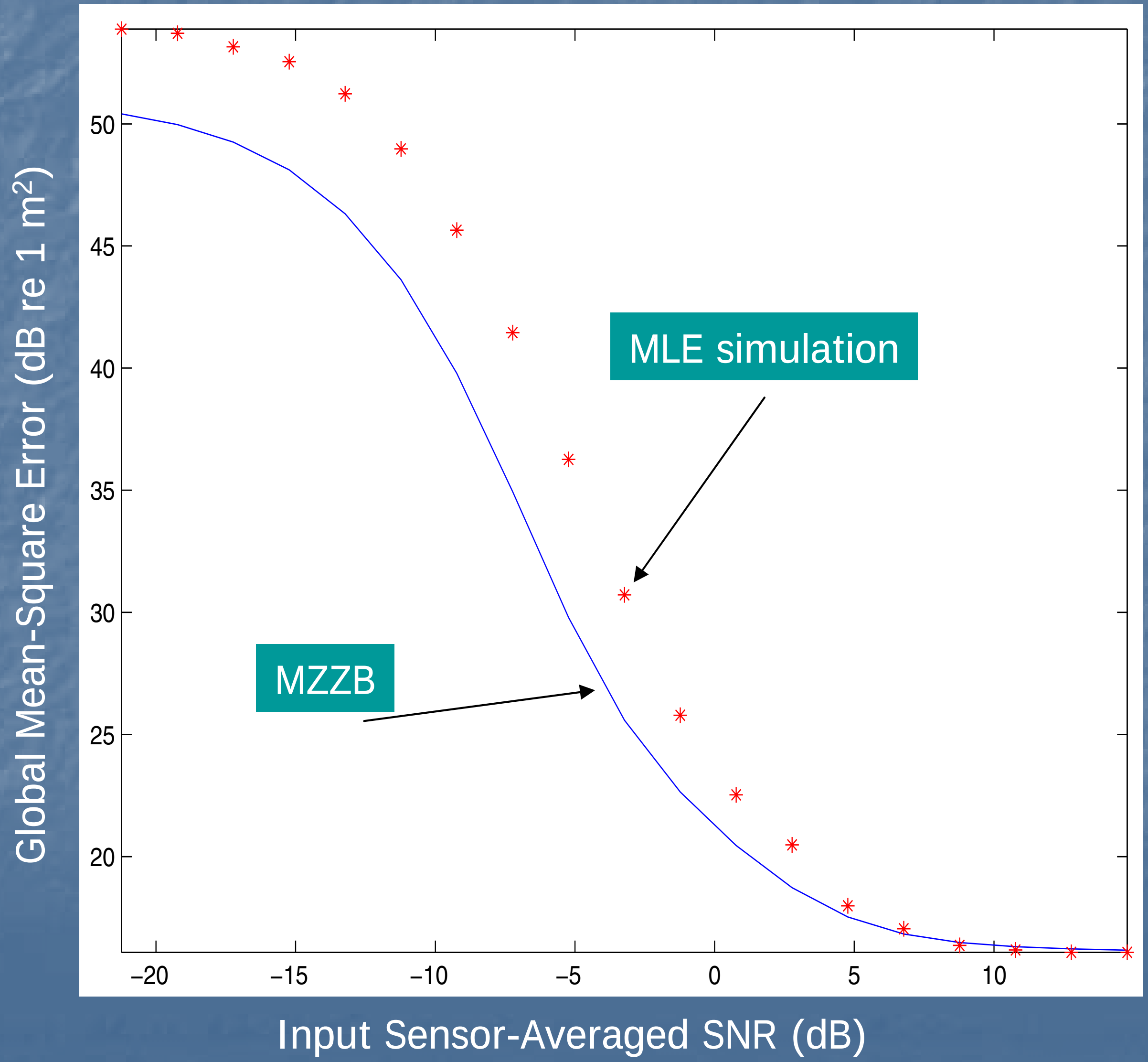


Example environmental model in SWellEX-3

Source frequency: 101 Hz

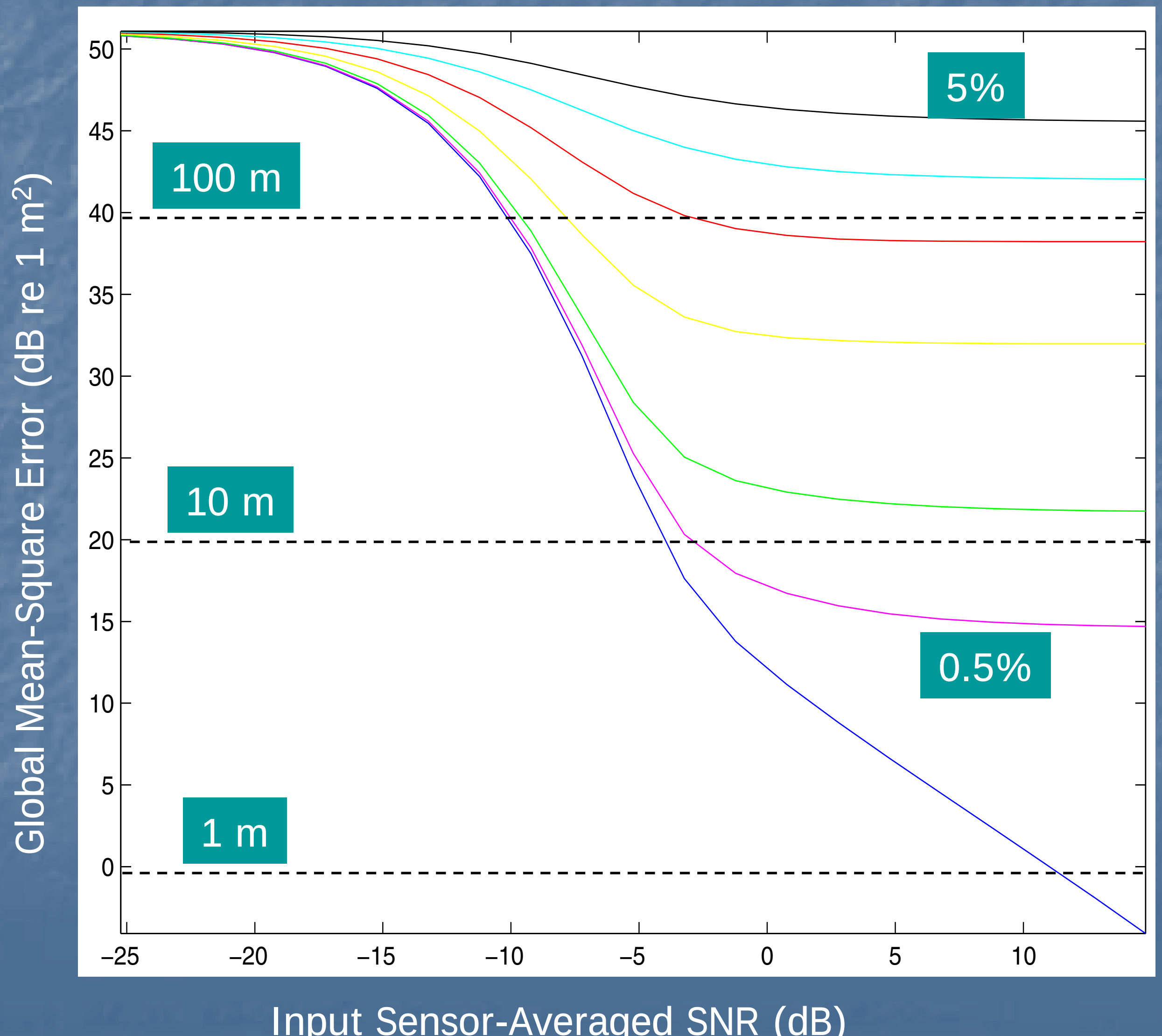
Model provided by Aaron Thode of Scripps Inst. Oceanography

Source range estimation with mismatched sediment wave-speeds



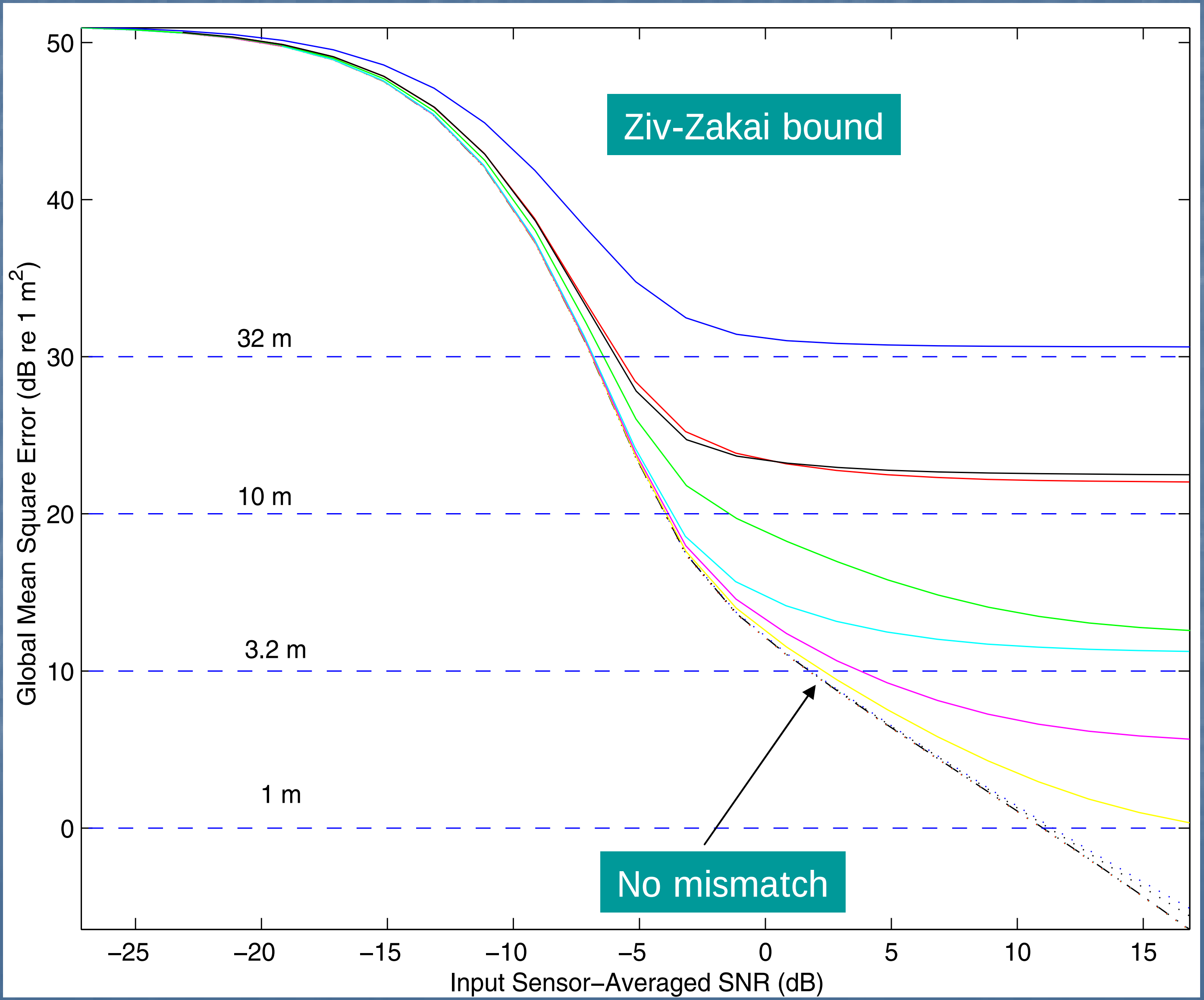
- Source range: 3500 – 4750 m
- Source depth: 60 m
- Sediment top wave-speed: 1550 m/s (true); 1572 m/s (modeled)
- Sediment bottom wave-speed: 1625 m/s (true); 1593 m/s (modeled)
- Above SNR = 5 dB, MLE bias = 5.7 m
- At SNR = 15 dB, bias predicted by ZZB = 6.4 m

Source range estimation with mismatched sediment wave-speeds (continued)



- Source range: 3500 – 4750 m
- Source depth: 60 m
- Sediment top wave-speed (m/s) – Modeled: 1572 True: (top to bottom) 1497; 1512; 1526; 1541; 1556; 1564; 1572

Source range estimation with 1% mismatch at individual bottom parameters



S1: Sediment 1
S2: Sediment 2
BM: Basement

All

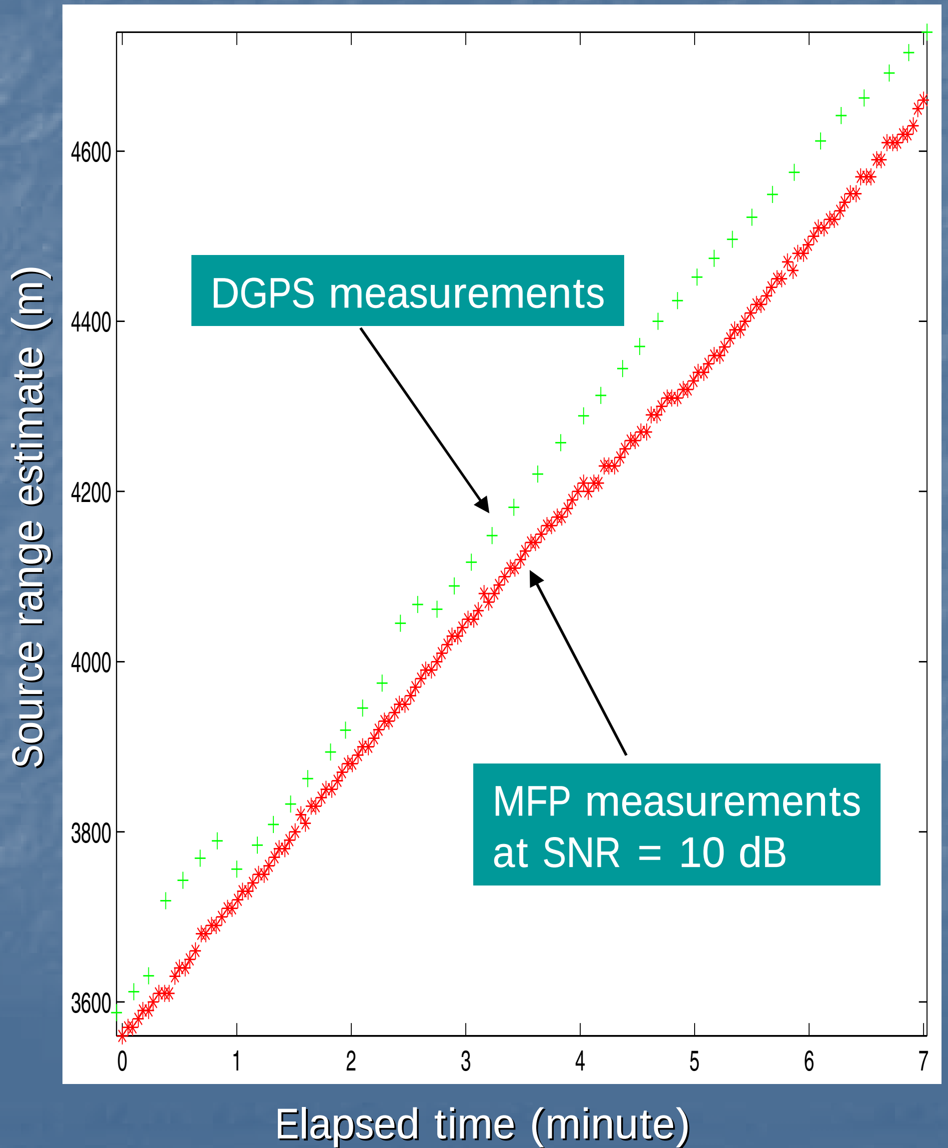
S1 bottom speed
S1 top speed

S2 top speed
S1 thickness

S2 bottom speed

S2 thickness
S1/S2 density /attenuation, BM properties

Example SWellEX-3 experimental data processing



Averaged offset: ~ 80 m

- DGPS accuracy: ± 5 m
- Offset from the GPS station to the towed source: ~ tens of meters
- Offset from the GPS station to the receiver array: small
- Source moving: 6 seconds => ~ 17 m
- Output correlation shows about one dB mismatch loss (Booth, *et al.*, JOE, 1996);

Given one dB correlation peak degradation, the modified ZZB predicts a bias of ~ 40 m

Data provided by Phil Schey of NAVY SPAWAR

A Ziv-Zakai type lower bound has been developed to analyze array processing performance under system model mismatch

- Incorporate the increase in mean-square error due to estimation bias at high SNR
- Include the threshold effect due to estimation ambiguity at low SNR
- Work for both spatially-white and correlated noise field
- Account for mismatch in both signal and noise propagation
- Generalization available to vector parameter estimation and stochastic mismatch

Further issues

- Simulate spatially-correlated noise application
- Investigate how the mismatch affects the threshold SNR

Further reference

W. Xu, A. B. Baggeroer, and K. L. Bell, “A bound on mean-square estimation error with background parameter mismatch,” to appear in *IEEE Trans. Information Theory*, April, 2004